Developing a New Dry Zenith Path Delay Model for Egypt

Farhan, M.*, Shaker, A. **, Saad, A. **, Mahmoud, S. ***, M. Rabah. ***

* *Faculty of Engineering, Kafer Elshekh University*
** *Shoubra faculty of Engineering, Benha University*

*** *National Research Institute of Astronomy and Geophysics*

Abstract

Natural atmospheric delay is one of the major sources of GPS errors that hinder using it in precise geodetic application. Global dry tropospheric models are derived using available Radiosonde data obtained from Europe and North America continents. The global atmosphere conditions used as constants in these models provide a broad approximation of the tropospheric conditions. Unfortunately, these models did not meet the local meteorological conditions of Egypt. In other words, they do not take into account the latitudinal and seasonal variations in the atmosphere**.** Besides, daily variation in temperature, pressure and relative humidity can lead to error in tropospheric delays obtained using the global tropospheric models especially in the height components. Hence, these models do not truly reflect the actual tropospheric effect.

With increasing the demands of utilizing GPS in precise geodetic applications, it was inevitably to have a local model to compute the zenith dry tropospheric delay. The current study submits a new local developed model to compute the dry zenith tropospheric delay. The new model is derived from ray tracing of meteorological data at 17 sites covering all Egyptian territory. The new developed model is a linear relationship with the measured surface pressure and temperature data. The tested data at the five locations shows a very good performance. The results confirm how the new local developed model can enhance the use of GPS in precise geodetic applications.

Keywords: GPS, atmospheric delay, global dry tropospheric models, dry zenith tropospheric delay, ray tracing

1. Introduction

The troposphere is the lower part of the earth's atmosphere, where the temperature decreases with the altitude, the thickness of the troposphere is not the same everywhere. It extends to a height of less than 9 km over the poles and to more than 16 km over the equator and extends from the sea to about 50 km (**Hofmanen- Wellenhof et al., 2001**). Tropospheric path delay is a major source of error in deep space tracking. However, the tropospheric–induced delay at tracking sites can be calibrated using measurements of Global Positioning System (GPS) satellites. The delay in radio signals caused by troposphere range from 2m at zenith to 20m at lower elevation angles (below 10 degrees) **(Dodo and Kamarudin , 2008**). The tropospheric delay consists of two components, dry and wet. Approximately 90% of tropospheric delay caused by refraction is due to dry component of troposphere; it is a function of elevation and altitude of the receiver, which depends on factors such as atmospheric temperature, pressure and relative humidity. It is not frequency dependent as in the case with the ionosphere and cannot be eliminated through linear combination of L1 and L2 observations **(Satirapod and Chalermwattanachai, 2005),**

2. Theoretical Background

The tropospheric delay consists of the dry and wet components. The dry component is a function of surface pressure and accounts for about 90 % while the wet component is a function of the distribution of water vapor in the atmosphere and accounts for 10% of the total delay (**Misra and Enge, 2001)**. Therefore, the tropospheric delay has been shown to be directly proportional to the refractive index and this is functionally expressed as (**Hoffman –Wellenhof et al. 2001**):

$$D^{trop} = \int (n-1)\, ds \qquad (1)$$

Expressing in term of refractivity,

$$D^{trop} = 10^{-6} \int N^{trop} ds \qquad (2)$$

Where:

N^{trop} =$10^{-6}(n-1)$ is the tropospheric refractivity and n is the refractive index.

The delay of GPS signal can be functionally expressed as the sum of dry and wet contributions to the total delay as follows:

$$D^{trop} = 10^{-6} \int_{path} N_d^{trop} ds + 10^{-6} \int_{path} N_w^{trop} ds \qquad (3)$$

The refractivity N^{trop} is expressed in terms of absolute temperature, partial pressure of the dry gases(P_d)and water vapor (e) in millibars as follows (**Mendes and Langely, 1994).**

$$N^{trop} = K_1 \frac{P_d}{T} \mathrm{Z}_{\mathrm{d}}^{-1} + k_2' \frac{\mathrm{e}}{\mathrm{T}} \mathrm{Z}_{\mathrm{w}}^{-1} + \mathrm{K}_3 \frac{\mathrm{e}}{T^2} \mathrm{Z}_{\mathrm{w}}^{-1} \qquad (4)$$

$$= N_d^{trop} + N_w^{trop} ,$$

Where:

P_d	is the partial pressure of the dry air in mbar
e	is the partial pressure of the water vapor in mbar
T	is the absolute temperature in Kelvin
Z_d^{-1}, Z_w^{-1}	are corrections for non- ideal gas behavior for the dry air and water vapor respectively, their values are very close to unity and can be found in (Owens,1967) [J$kg^{-1}k^{-1}$].
K_i	Empirically determined coefficients (**Mendes, 1999**), [K h Pa^{-1}]
R_d, R_w	Gas constant respectively for dry component and wet component [J$kg^{-1}k^{-1}$]

$$k_2' = k_2 - k_1 \frac{R_d}{R_w} \tag{5}$$

Where the constant coefficients K_1, k_2' and K_3 are empirically determined ,Z_d and Z_w are factor for dry air and water vapor, the frequently used sets of refractivity constant are given in table (1).

Reference	**K_1 [K hPa^{-1}]**	**k_2[K hPa^{-1}]**	**$k_3 10^5$[K hPa^{-1}]**	**k_2'[K hPa^{-1}]**
(Boudouris,1963)	77.59 ± 0.08	72 ± 11	3.75 ± 0.03	24 ±11
(Smith and Weintraub,1953)	77.61 ± 0.01	72 ± 9	3.75 ± 0.03	24 ± 9
Thayer,1974)	77.60 ± 0.01	64.79 ± 0.08	3.776 ± 0.004	17 ± 10

Table (1): Frequently used sets of refractivity constants

The integral of the dry refractivity (dry delay) is difficult to determine from this definition because the dry equation is valid for the total pressure of the dry air only. Fortunately, the first two terms in equation (4) have the same temperature dependence and using the gas law for each gas "i" with compressibility Z_i , one can get (**Davis et al., 1985**):

$$P_i = \rho_i \frac{R}{m_i} T Z_i \tag{6}$$

Where:

ρ_i	is the density
m_i	is the molar mass
R	is the universal gas constant (8314.34±0.35J/Kmol K),

Equation (4) can be written as:

$$N^{trop} = K_1 \frac{R}{m_d} \rho + K_2' \frac{m_d}{m_w} \frac{e}{T} Z_w^{-1} + K_3 \frac{e}{T^2} Z_w^{-1} \tag{7}$$

Where: ρ is the total density, m_d, m_w are the molar masses of dry (28.9644± 0.0014 kg/ kmol) and wet (18.0152 kg/kmol) air, respectively (**Elgered, 1993)**.

Since the dry delay δT_d is our main concern here, it is expressed as:

$$\delta T_d = 10^{-6} \int K_1 \frac{R}{m_d} \rho \, ds \tag{8}$$

3-1 Dry Zenith Path Delay

Using equation (8), the dry delay in the zenith direction δT_d^Z is obtained by replacing the path elements ds with the vertical elements *dh* in equation (8) as:

$$\delta T_d^Z = 10^{-6} \int K_1 \frac{R}{m_d} \rho \, dh$$

$$= 10^{-6} \int N_d^{trop} \, dh \tag{9}$$

4 Data Under consideration

- Seventeen point distributed all over Egypt were selected to be used in the design of a new model, as seen in figure (2).

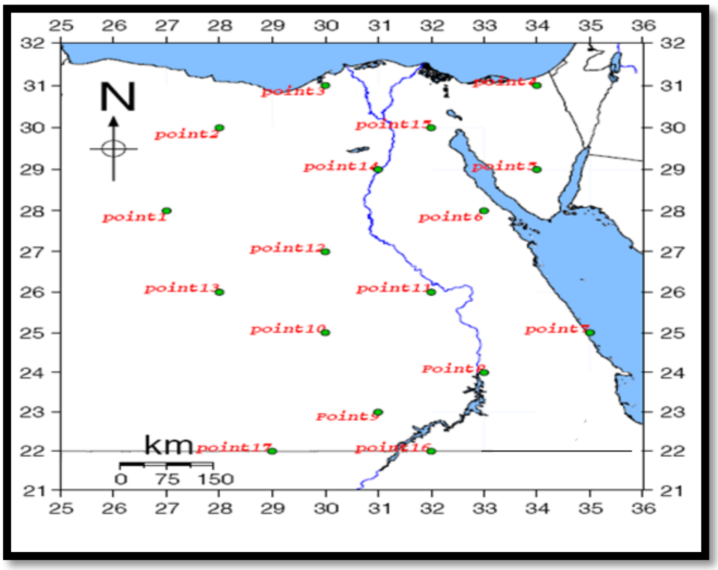

Figure (2): Seventeen data points distributed all over Egypt

- For every station, the day of year, average values of meteorological parameters are used.
- The Mass-Spectrometer –Incoherent –Scatter (MSIS) model, which describes the neutral temperature, densities in earth's atmosphere from ground to thermospheric height (**Hedin,(1991).**, is used to profile the temperature and pressure that is needed for the ray tracing analysis. Ray tracing is the process of determining the path of an electromagnetic signal, based on geometric optics theory applied over a series of thin spherical shells, concentric with the earth, and within which a constant refractivity is assumed

5- Practical Procedure

In order to find out the relation between the tropospheric dry zenith delay an surface pressure the following procedure is applied:

- An average value of three different period of times (at one, eight and thirteen clock) is taken for the evaluation.
- The troposphere layer is divided into different elements. From 0 to 1km by interval 100 m, from 1km to 5 km by interval 200 m and from 5km to 50 km by interval 1km as shown the figure (3).

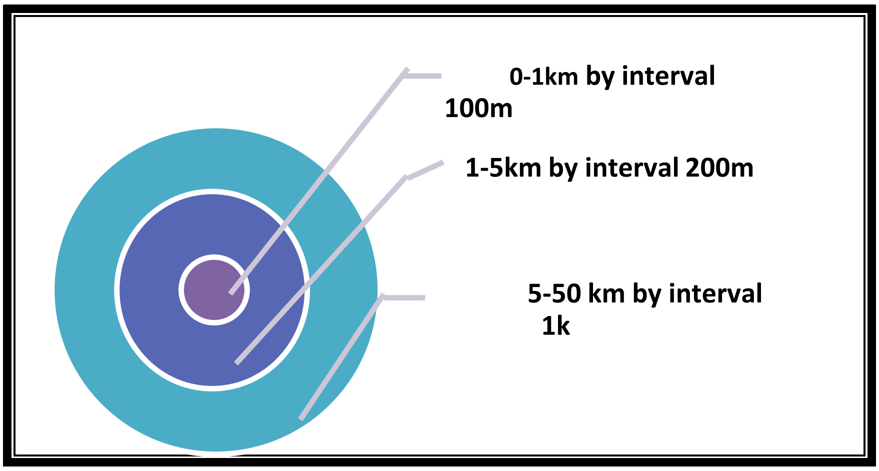

Figure (3): different elements of troposphere

- Dry refractivity is calculated from the temperature and pressure values at each layer in three cases (at one, eight and 13 O'clock) using the first part of equation (4), then the ray tracing is carried out numerically by integrating equation (9), the results are given in table (2), figure (4).

Point	T (K)	P (HP)	δT_d^Z (m)	P/T (HP/K)
1	294.8	1012.958769	2.309857756	3.436088089
2	293.7	1014.237613	2.312917773	3.453311588
3	293	1014.343538	2.314309003	3.461923338
4	292.9	1014.838139	2.314001264	3.464793921
5	294.1	1013.086222	2.310920197	3.444699838
6	294.7	1012.61516	2.309310887	3.436088089
7	296.3	1010.457917	2.304232913	3.41025284
8	296.8	1009.607076	2.302786327	3.40164109
9	297.3	1008.747623	2.301515385	3.393029341
10	296.3	1010.457917	2.304688006	3.41025284
11	295.8	1011.300146	2.306134814	3.41886459
12	295.3	1012.133763	2.307990716	3.427476339
13	295.9	1011.642032	2.306484766	3.41886459
14	294.2	1013.430692	2.311212232	3.444699838
15	293.6	1013.892282	2.312689224	3.453311588
16	297.7	1008.39569	2.29982321	3.387288174
17	297.7	1008.39569	2.300157569	3.387288174

Table (2): The Zenith dry delay and surface pressure

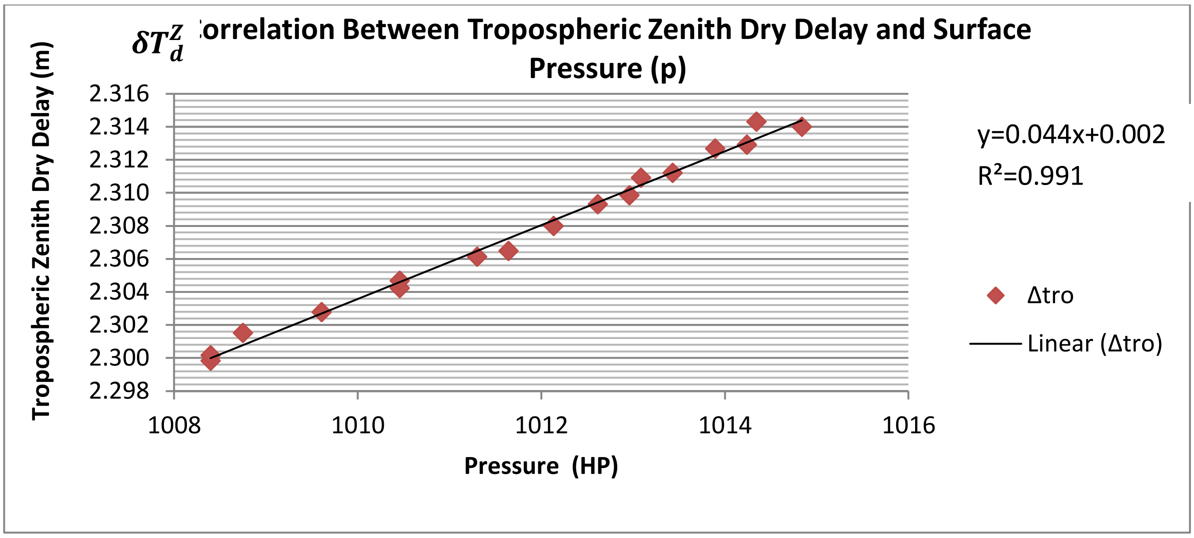

Figure (4): Correlation between Zenith dry delay and surface pressure

- Least square fitting technique is applied to linearly fit the data.
- The parameters of the linear relation between the zenith dry delay and the surface pressure and the new local model is given as

$$\delta T_d^Z = a * P + b \tag{10}$$

With: *a* = 0.002, *b* = 0.044

Also the refractive at any point in a dry atmosphere depends on the pressure and temperature (the ratio P/T in Equation (4)). Thus a new linear relation between zenith dry delay and the surface pressure / temperature is calculated figure (5).

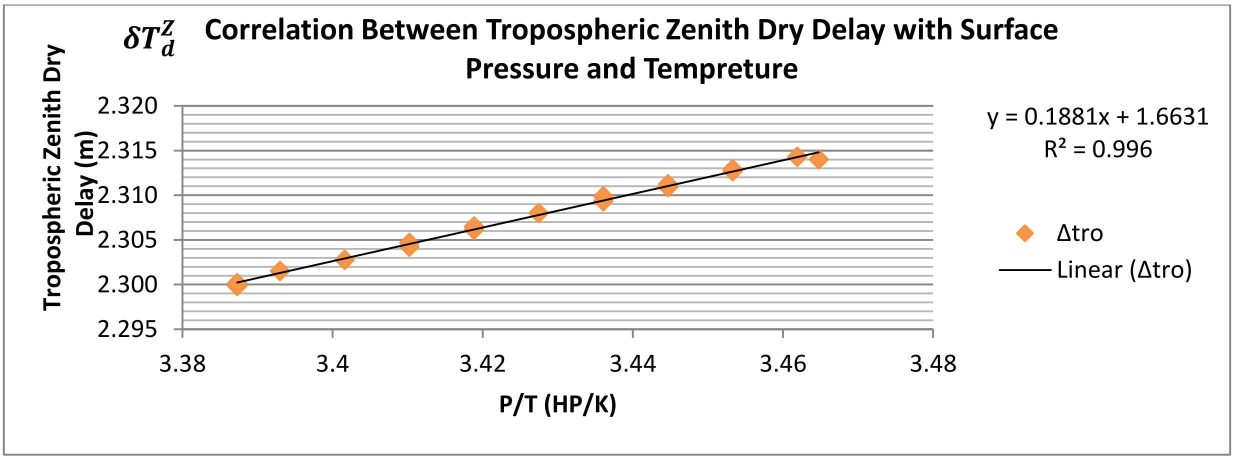

Figure (5): The linear relationship between Zenith dry delay with surface pressure and temperature

Figure (5) clearly indicates that the linearity between the ray traced zenith delay with surface pressure and temperature are very strong compared to the stand alone surface pressure as depicted in figure (4). The correlation coefficient was found to be 0.996.

The second new local model takes the form:

$$\boldsymbol{\delta T_d^Z = a * \frac{P}{T} + b} \tag{11}$$

With *a* = 0.188 and *b* = 1.663 as constant parameters.

5 Verifying the Developed Model

To verify the developed model, A performance evaluation of the new model was done against two of the precise global tropospheric models, namely Hopfield, Saastamoinen and one local model namely, Mousa model (Mousa et al, 2003

The verification was performed over five points distributed over Egypt. The five test locations are Salum, Helwan, Safaga, Alis (Aswan) and Shlatin. The location of the five test locations are illustrated in figure (7). For every test location, the dry zenith tropospheric ray tracing is computed for the new model and the two global and one local tropospheric models. The computed values are tabulated in table (4) and the differences were drawn in figure (8).

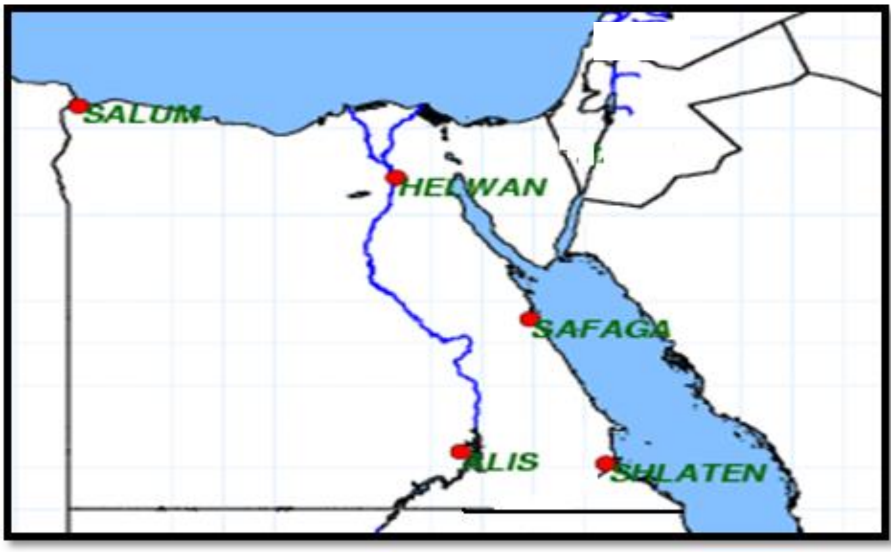

Figure (7): Five-check points distribution

Testing points	**Total Dry Zenith delay (m)**	**Hopfield**	**Saastamoinen**	**Mousa et al., 2003**	**New model**
		Difference(m)	**Difference(m)**	**Difference(m)**	**Difference(m)**
SALUM	**2.315402**	2.319236	2.305592	2.31679377	2.31534
		0.00383	**-0.00981**	**0.00139**	**-0.00006**
HELWAN	**2.312399**	2.316016	2.3023194	2.31355649	2.3126400
		0.00362	**-0.01008**	**0.00157**	**-0.00024**
SAFG	**2.306914**	2.309063	2.295249	2.30656011	2.306701
		0.00215	**-0.01166**	**0.00035**	**-0.00021**
SHLATIN	**2.3013241**	2.303012	2.289087	2.30046331	2.301301
		0.00169	**-0.0122**	**0.00086**	**-0.00002**
ALIS	**2.3018634**	2.304176	2.2902527	2.30161691	2.301841
		0.00231	**-0.0116**	**0.00025**	**-0.00002**

Table (4): The performance of developed new model against Global, Local models

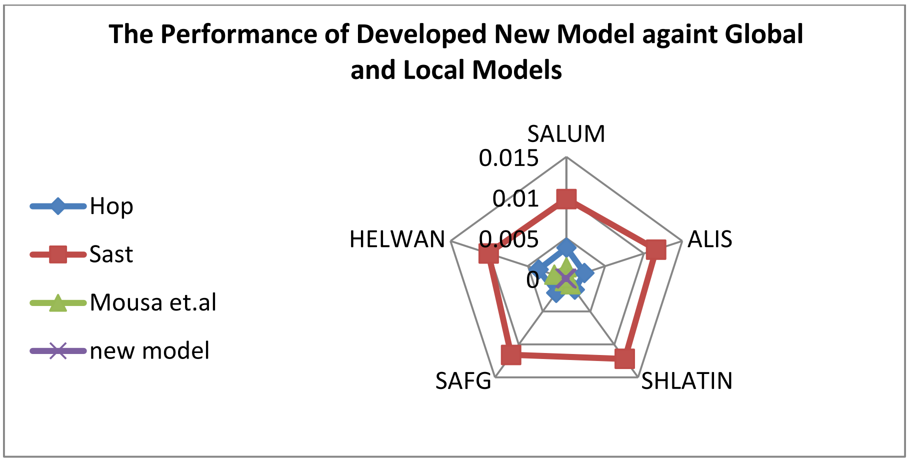

Figure (8): The performance of developed new model against Global and Local Models

From the differences in table (4), new model yield less differences compared with another models.

Conclusions

The current study submits a new local developed model to compute the tropospheric dry zenith delay. The new derived model used the ray tracing of meteorological data at 17 sites covering all Egyptian territory. The new developed model have a linear relationship with the measured surface pressure or measured surface pressure and temperature data. The tested data at the five locations shows a very good performance, less than one mm. The results confirm how the new local developed model can enhance the use of GPS in precise geodetic applications.

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